Analysis of Polarization Mode Dispersion Effect on Quantum State Decoherence in Fiber-based Optical Quantum Communication

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Abstract: **The implementation of quantum communication protocols in long optical fibers is limited by several decoherence mechanisms. In this contribution we review this mechanisms and analysis their effect on the quantum states. Because of asymmetry in the real fibers, the two orthogonal polarization modes are propagated at different phase and group velocities. The difference in group velocities results in Polarization Mode Dispersion (PMD). It is created by random fluctuations of the residual birefringence in optical fibers, such that the State of Polarization (SOP) of an optical signal will turn randomly over time, in an unpredictable way. In optical quantum communication, quantum state measurement is necessary. The bottleneck for communication between far apart nodes is the increasing of the error probability with the length of the channel connecting the nodes. For an optical fiber, the value of absorption and depolarization of a photon (i.e., the qubit) increases exponentially with the length of the fiber.**

Keywords: Quantum state, polarization, quantum communication, group delay, birefringence, decoherence, PMD

I. Introduction

The orthogonally polarized modes are preserved only for an ideal single-mode fiber with a perfectly cylindrical core of uniform diameter. Real fibers expose considerable variation in the shape of their core along the fiber length. They may also confront nonuniform stress such that the cylindrical symmetry of the fiber is broken. Degeneracy between the orthogonally polarized fiber modes is lost because of these factors, and the fiber gets birefringence. The degree of modal birefringence is defined by [1],

$$B_m = \left| n_x - n_y \right| \quad (1)$$

Where n_x and n_y are the mode indices for the orthogonally polarized fiber modes. Birefringence results to a periodic power exchange between the two polarization components. A characteristic of optical fiber used to calculate the fiber's ability to maintain polarization is called beat length. The beat length describes the length required for the polarization to rotate 360 degrees. For a given wavelength, it is inversely proportional to the fiber's birefringence. It is given by

$$L_B = \lambda / B_m \quad (2)$$

Typically, $B_m = 10^{-7}$, and $L_B = 10\,\text{m}$ for $\lambda = 1\,\mu\text{m}$. From a physical viewpoint, linearly polarized light preserves linearly polarized only when it is polarized along one of the principal axes. Otherwise, its state of polarization turns along the fiber length from linear to elliptical, and then back to linear, in a periodic manner over the length L_B. periodically change in the state of polarization is occurred for a fiber of constant birefringence B. in conventional single-mode fibers, birefringence is not fix along the fiber but changes randomly, both in magnitude and direction, because of variations in the core shape (elliptical rather than circular) and the anisotropic stress acting on the core [2]. As a result, the polarization state of light launched into the fiber quickly changes to the arbitrary polarization state. Moreover, different frequency components of a pulse obtain different polarization states, lead to pulse broadening. This problem is called polarization mode dispersion (PMD). PMD is a limiting factor for optical communication systems operating at high bit rates [3].

The particular states of polarization in each point of a fiber are determined by both the state of polarization launched into a fiber and the orientation and type of fiber birefringence. In quantum communications applications these factors are very important [4].

II. The Fiber-based Quantum Channel

An important component for realizing quantum communication schemes over long distance is the ability to transmit the qubits, in our case photons, in a convenient way. The most suitable medium is an optical fiber, which can be installed like any other cable and works in a very stable way. For the transmission of photons only single-mode optical fibers can be used.

However, in the case of qubits represented by polarized photons, only single-mode fibers are suitable for transmitting the photons. In multi-mode fibers, the different propagation of several modes immediately leads to depolarization of the photons. Polarization-preserving fibers are single-mode and can preserve two distinct

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polarization directions, yet they are not suitable because they do not allow the transmission of a general polarization state, which occurs for the general state of a qubit.

The most critical measure of a single-mode fiber is the cutoff wavelength λ_{cutoff}. This is the minimal wavelength at which the light can only propagate in a single mode. λ_{cutoff} is defined by the diameter of the core. Another important parameter is the damping of the fiber due to absorption and other losses. This value is usually given in dB/km. A parameter interesting for coupling light in and out of fibers is the numerical aperture (NA) of a bare fiber, which is governed by the ratio of the core and coating refractive indices.

A. *Polarization Effects in Fiber Transmission*

The unavoidable geometrical curvature of a laid-out optical fiber leads to stresses which induces slight birefringence in the glass. Generally, these birefringences are randomly distributed, and will hence produce an arbitrary transformation of the polarization of the photons. For suitable distances, where depolarization does not occur, the induced polarization transformation remains unitary and hence can be fully compensated using polarization controllers.

The transformation will change over time, since the mechanical stress in the fiber varies because of temperature changes or mechanical changes [5]. In a practical system, the fiber must be kept under constant mechanical and thermal conditions, in order to enhance the stability of the polarization transformation. Some test measurements show, that for a fiber of a few meters length which is taped onto an optical table, the stability of the polarization can better than 1° over a time of 24h. A 500 m cable installed in the basement of building shows drifts of the polarization in the range of 1° in 2 h.

The polarization is best viewed in terms of the Poincare sphere. A unitary transformation, such as the effects of the fiber, will simply rotate the whole sphere by a certain angle around an axis. Clearly, in order to obtain a useful transmission of the qubits, this transformation has to be compensated.

In principle, this transformation could be corrected with polarization optics, such as wave plates. In terms of the Poincare sphere, a wave plate is an element which has an axis sitting in the equator plane and will perform a rotation of the whole sphere around the axis by a certain fraction of 2π, e.g. a half-wave plate performs a rotation of $2\pi/2$. The axis of the wave plate is adjusted by a simple rotation. So by combining several wave plates, any transformation of the sphere can be accomplished and the qubits are received in their correct state after the fiber transmission.

The best way to control the polarization transformation in a fiber is to use the effects of the actual fiber instead of external optics, by deliberately inducing birefringence in the fiber.

B. *Depolarization of the photons in fibers*

The single-mode fiber will also induce depolarizing effects on the transmitted photons, which unfortunately cannot be compensated as easily as the polarization rotation caused by birefringence. We will only discuss the two most obvious effects.

Polarization mode dispersion (PMD) is the difference in the propagation velocity of two orthogonal polarization modes propagating in the fiber. It originates from the random birefringent segments of the fiber and shows statistics similar to that of random walk of particles. Therefore PMD is given in units of time per square root of the fiber length, and typical values of a modern telecom fibers are in the range of 0.1 – 0.5ps/√km [6].

PMD leads to a depolarization of the light when the separation of the polarization modes is larger than the coherence time of the photons. For instance, for down-conversion photons with $\Delta\lambda$ =1.5 nm (corresponds to $\Delta\lambda_{FWHM}$=3.5nm), the coherence time is τ_{cohr}=220 fs. Therefore, even for the best fiber with a PMD of 100 fs/√km, one might run into problems of depolarization at a fiber length of ≈ 10 km.

A simple model for calculating the effect of PMD on the visibility of polarization is to assume that the overlap of the two orthogonal polarization wave packets reduces as a Gaussian function depending on the ratio τ_{pmd}/τ_{chor} in the form:

$$V = Ae^{\frac{-\tau_{PMD}^2}{2\tau_{chor}^2}} \tag{3}$$

where A is an amplitude factor (allowing for a non-ideal initial polarization) and τ_{PMD} =PMD×$\sqrt{L_{fiber}}$ depends on the PMD value and the length of the fiber. The calculated visibility of polarization for a fiber of up to 30 km length is shown in Figure (1) for down-conversion photons travelling through a single-mode fiber. The photons were polarized before coupling into the fiber and analyzed after the fiber with a two channel polarization analyzer and photon counters.

The polarization visibility has been calculated for PMD value of about 25 fs/√km, which is much less than the value for telecom fibers which is at best 100 fs/√km, and for PMD value of about 100 fs/√km.

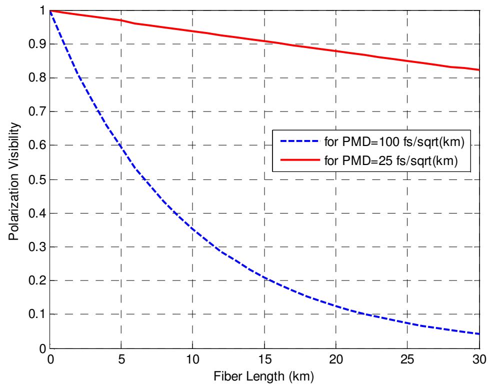

Figure 1. Visibility of the polarization of down-conversion photons after travelling through a single-mode fiber

Chromatic dispersion (CD) can also impose a problem on the transmission of photons with a finite bandwidth, as the different spectral components have different group velocities due to the dispersion relation of the medium. Depending on the fiber length and spectral width of the photons $\Delta\lambda$, CD could stretch the spectral components of the wave packet. However, CD is only important for the timing of the photons, i.e. in an interferometer. The polarization of the photons is not lost. For fused silica, such as a fiber, the slope of the refractive index centered at λ=800 nm is δn(800nm)≈Δn/Δλ=1.71×10−5 nm-1. The temporal broadening of a wave packet depending on dn is dτ = L_{fiber} dn/c with L_{fiber} being the length of the fiber and c the vacuum speed of light. This expression can be used for an estimation of the effect of CD by expressing dn ≈ δnΔλ which leads to $\pm\Delta\tau_{cd} \approx L_{fiber}\delta n\Delta\lambda/c$. For photons with a spectral width of Δλ=1.5 nm and a fiber length of 1000 m, the CD induced shift would be ±85 ps, and would therefore strongly broaden the wave packet. However, CD is static and can be easily compensated externally e.g. by compensation prisms or chirped mirrors.

C. *Coherence Time*

In vacuum, the dispersion relation has the simple form: c=λω/2π, where c is the vacuum speed of light, λ is the wavelength, and ω is the angular frequency of the wave. If the spectral distribution of a light signal is Gaussian with a width of Δω, then the conjugated temporal distribution of the wave packet is also a Gaussian, with a width: Δω · Δτ ≥ 1. This Δτ can be identified with the coherence time, or longitudinal coherence, of the associated wave packet. By expressing Δω by Δλ via the derivative of the dispersion relation given above, the coherence time has the form:

$$\tau_{cohr} = \frac{\lambda^2}{2\pi c\,\Delta\lambda} \tag{4}$$

Here λ is the central wavelength, Δλ is the width of the spectral distribution and c is the vacuum speed of light. For instance, for down-conversion photons with λ=788nm and Δλ=1.5nm (corresponds to $\Delta\lambda_{FWHM}$=3.5nm), the coherence time is τ_{cohr} = 220 fs. Figure (2) shows decoherence time vs. spectral width.

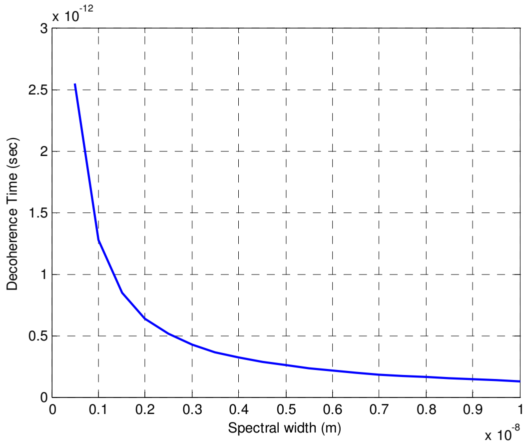

Figure 2. Decoherence time vs. spectral width

III. Decoherence in the Quantum Communication

In Jones space polarization states are depicted by the two-dimensional complex column "ket" vector $|s\rangle = \begin{pmatrix} s_x \\ s_y \end{pmatrix}$. The "bra" $\langle s| = (s_x^*, s_y^*)$ is the corresponding conjugate row vector. The corresponding Stokes vector $\hat{S} = (S_1, S_2, S_3)$ is specified by $\hat{S} = \langle S|\vec{\sigma}|S\rangle \big/ \langle S|S\rangle$ where $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is the Pauli matrix vector whose three components are the Pauli matrices.

$$\sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{5}$$

They are defined in "right circular" Stokes space: The Stokes parameter $S_3 = \langle S|\vec{\sigma}_3|S\rangle \big/ \langle S|S\rangle$ is positive and equal to one. The Stokes vector $\hat{S}$ is real (since $\vec{\sigma}$ is Hermitian), and it is of unit length. Thus its end point locates on the surface of a unit sphere, called Poincaré sphere, a three dimensional mapping of each polarization state. The three Stokes parameters S_1, S_2 and S_3 or the corresponding angles of the Poincaré sphere 2Ψ and 2χ (0°< 2Ψ <360°, 90°< 2χ <90°) describe the state of polarization completely.

The expression of a PDL element (with the direction of the least attenuated state of polarization $\hat{e} \equiv (e_1, e_2, e_3)$, the largest transmission coefficient T_{max} and the differential loss vector $\vec{\alpha} \equiv \alpha.\hat{e}$) on the input Jones vector $|s\rangle$ is defined by,

$$|t\rangle = \sqrt{T_{max}}\exp\left(-\frac{\alpha}{2}\right)\exp\left(\frac{\vec{\alpha}}{2}\vec{\sigma}\right)|s\rangle \tag{6}$$

Using the relation exp(A) = 1 + A/1! + A2/2! + A3/3! + … (which is valid for any complex 2×2 matrix A, the matrix 1 being the 2×2 identity matrix) and the connection between the Pauli matrices ($\sigma_i\sigma_j = i\sigma_k$ for any cyclic permutation of (i, j, k)), we find the more manageable equation

$$|t\rangle = \sqrt{T_{max}}\exp\left(-\frac{\alpha}{2}\right)\left[\cosh(\alpha/2)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \sinh(\alpha/2)\begin{pmatrix} e_1 & e_2 - ie_3 \\ e_2 + ie_3 & -e_1 \end{pmatrix}\right]|s\rangle \tag{7}$$

If the input SOP $|s\rangle$ does not depend on frequency, the output SOP $|t\rangle$ is neither frequency-dependant.

Optical fibers used in today's networks have neglected PDL values compared to the PDL of components such as optical amplifiers or couplers. Thus a fiber can be modeled without PDL as a concatenation of N birefringent elements (with the frequency-independent DGD Δt_j of the jth element) with random orientations of the slowest polarization states $\hat{e}_j \equiv (e_{j1}, e_{j2}, e_{j3})$. The impact of a fiber on the input Jones vector s can therefore be described by,

$$|t\rangle(\omega) = \exp\left(-i\frac{\Delta\tau_N}{2}\omega\vec{e}_N.\vec{\sigma}\right).\exp\left(-i\frac{\Delta\tau_{N-1}}{2}\omega\vec{e}_{N-1}.\vec{\sigma}\right) \ldots \exp\left(-i\frac{\Delta\tau_1}{2}\omega\vec{e}_1.\vec{\sigma}\right)|s\rangle \tag{8}$$

Similar to the procedure leading to eq.(3) the transmission matrix of element j can be transformed to

$$\exp\left(-i\frac{\Delta\tau_j}{2}\omega\vec{e}_j.\vec{\sigma}\right) = \cos(\omega\Delta\tau_j/2)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - i\sin(\omega\Delta\tau_j/2)\begin{pmatrix} e_{j1} & e_{j2} - ie_{j3} \\ e_{j2} + e_{j3} & -e_{j1} \end{pmatrix} \tag{9}$$

Fortunately, polarization effects do not accumulate in long fiber spans in a linear fashion. Instead, because of random variations in the perturbations along a fiber span, the effects of one section of a fiber span may either add to or subtract from the effects of another section. As a result, PMD in long fiber spans accumulates in a random-walk-like process that leads to a square root of length.

Depolarization, i.e. decoherence in the polarization domain, severely limits potential applications of quantum cryptography [7]. Indeed, coding the qubit in polarization becomes clearly unpractical, while coding the qubit in the phase is no better because phase decoding requires interferometers and interferences are sensitive to polarization. One possible way out is to limit the width of the optical spectrum, so that the integral in (3) is dominated by the central frequency and the output light remains polarized. But even so, polarization fluctuations would impose active feedbacks.

IV. SIMULATION RESULTS AND DISCUSSION

In this section, the effects of fiber based channel for quantum communication will be analyzed. These effects are divided in two parts. First part is the photon loss and second part is related to dispersion in fiber channel. Consider a single mode fiber with core width of 4.15 μm and refractive index of 1.45213 and cladding width of 58.35 μm and refractive index of 1.44692.the total length of used fiber is 50 km. Its photon loss is 0.2dB/km. The used wavelength is 1.55 μm. in the first stage of simulation, photon loss due to material and bending losses are presented. Figure (3) and (4) show these losses. From these figures we can see that the total photon loss is about 0.2dB/km for wavelength 1.55 μm if there is not any micro bending.

Table 1: Parameters of fiber

No.	Parameter	Value
1.	Core width	4.15 μm
2.	Refractive index of core	1.45213
3.	Cladding width	58.35 μm
4.	Refractive index of cladding	1.44692
5.	Fiber length	50 km
6.	Loss	0.2 dB/km
7.	Wavelength	1.55 μm

A continuous wave laser with 0 dBm power in wavelength of 1.55 μm was used. The power spectrum for the input and output of fiber are shown in figure (5) and (6). As shown in these figures, the total loss in the channel is 10 dB. Although, this loss is not very important in quantum communication, but it should not be very big so any photon cannot arrive in the output of fiber channel. This photon loss is important in classical communication.

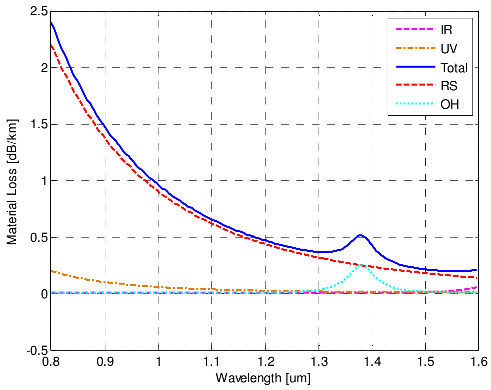

Figure 3. Material loss of fiber based channel

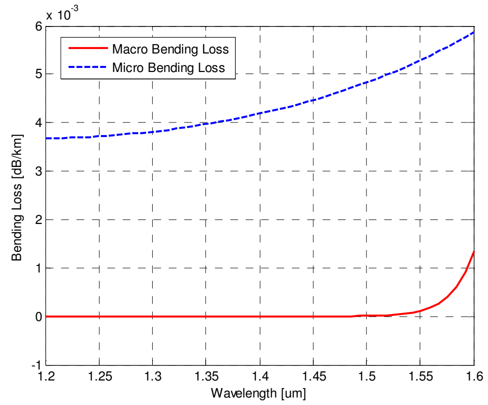

Figure 4. Bending loss of fiber based channel

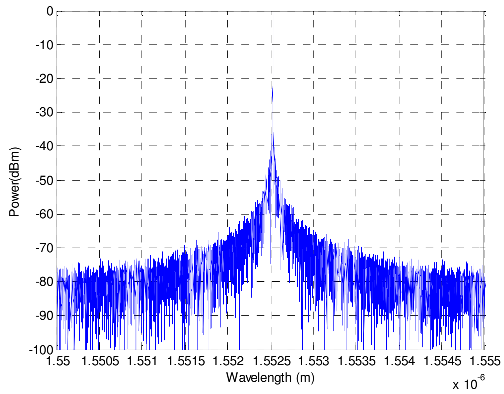

Figure 5. Power spectrum in the input of fiber channel

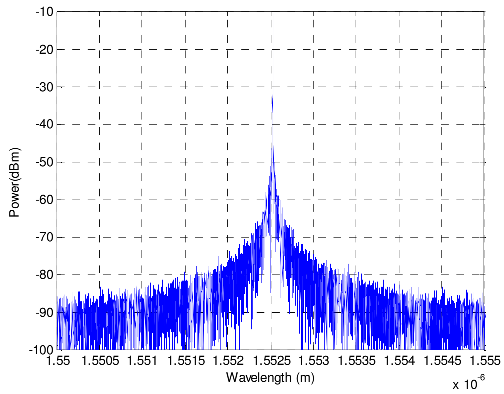

Figure 6. Power spectrum in the output of fiber channel

The detectors in quantum communication are single photon detectors, in the other word, if only one photon passes through channel safely, it is enough for quantum communication. The word 'safely' is very significant. The quantum state of photon must not change in the process of quantum communication. This means that, the fiber based channel must not have any destructive effects on quantum state of photon. The quantum state of photon is encoded in its polarization. The polarization state of photon in the input of fiber channel is shown in figures (7) and (8). As shown in these figures, the photon in the input of fiber channel has linear horizontal polarization. Because the information in quantum communication is encoded in the polarization of photon, state must not change in the transmission time, but due to the fact that the

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environmental of transmission channel affects the state of travelling photon and change it. As mentioned above, this effect entitled quantum decoherence. When the photon with horizontal polarization is passed through ideal channel without any environmental effects, the power spectrum of output photon must only exist in x direction and the y component must be zero. Because of birefringence effect in fiber, some dispersion such as material dispersion, waveguide dispersion and polarization mode dispersion affect on the photon state in the channel. Figure (9) shows dispersion vs. wavelength for used fiber. Another important parameter is group delay of transmission of photon modes while travel through the fiber. This parameter shows in figure (10).

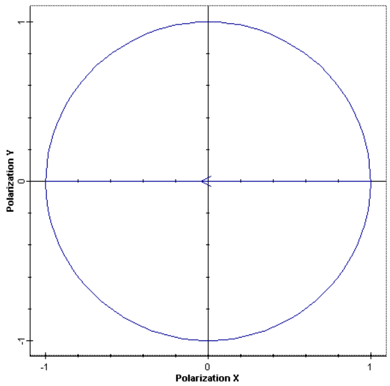

Figure 7. Polarization state of photon in the input of fiber channel

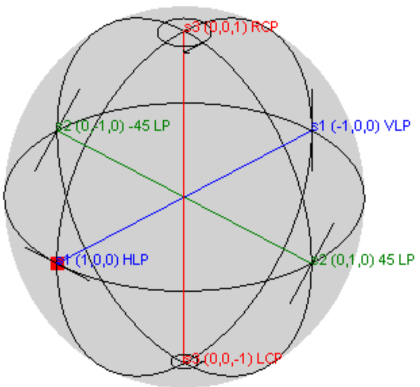

Figure 8. Polarization state of photon in the input of fiber channel on the Poincare sphere

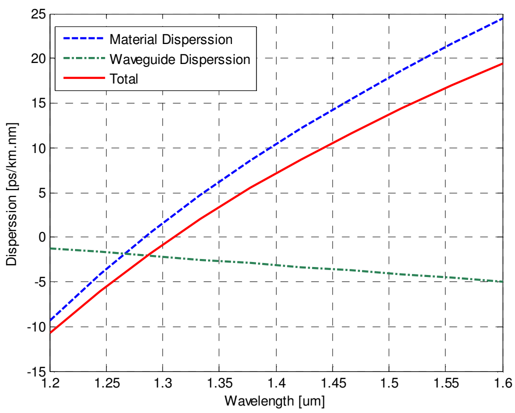

Figure 9. Material and waveguide dispersion

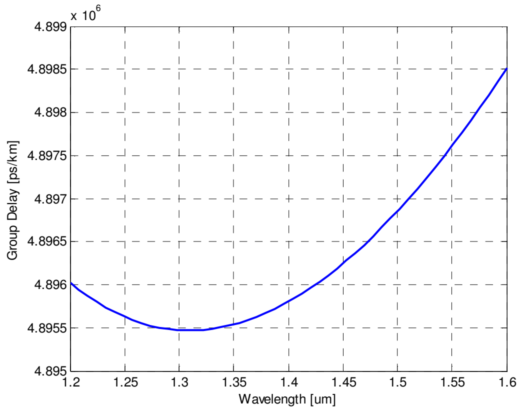

Figure 10. Group delay of transmission of photon modes

As mentioned above, we expect that the power spectrum of output photon only exists in x direction and the y component must be zero when horizontally polarized photon is passed through ideal channel without any environmental effects. In birefringence channel, the state of photon is changed and consequently the polarization state of output photon maybe is not horizontal and information encoded in polarization lost. Figures (11) and (12) show the x and y-polarized component of output photon. As shown in these figures, the polarization state has changed and some power goes to the y-polarized component.

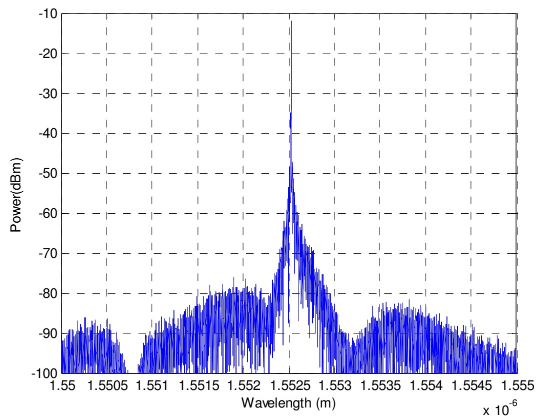

Figure 11. power spectrum of x-polarized component of output photon state

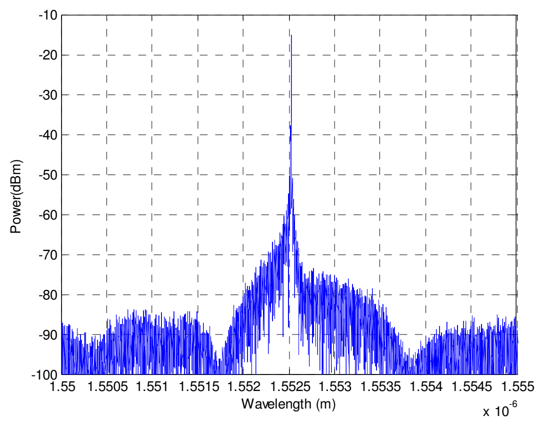

Figure 12. power spectrum of y-polarized component of output photon state

Polarization mode dispersion is most important parameter that can affect on quantum state of photon. Simulation results of PMD for used fiber is presented in figure (13). The mean value of first order of PMD is 0.344ps/km0.5 and its RMS is 0.354ps/km0.5.

Because of PMD effect, the quantum state (polarization state) of photon is destroyed along of fiber channel. This phenomenon is illustrated in figures (14) to (15) for different PMD value. As shown in these figures, the quantum state of input photon has been changed along the fiber channel and quantum information has been lost. According to figures (7) and (8), the polarization state of input photon was horizontally, but it is changed to elliptical polarized after passing through fiber channel. Due to PMD, both polarization form and rotation direction of polarization have been changed (figures (14) and (15)). Therefore, fiber-based channel restricts quantum communication for long distance application and for overcome this problem, fiber channel should be divided to smaller segments and some polarization optic components should be used in the end of fiber channel.

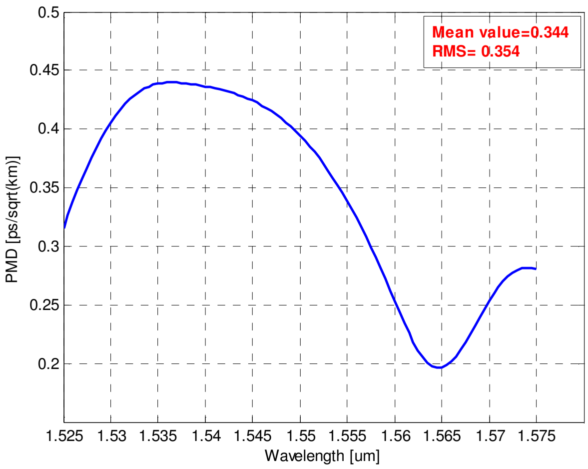

Figure 13. Polarization mode dispersion of fiber channel

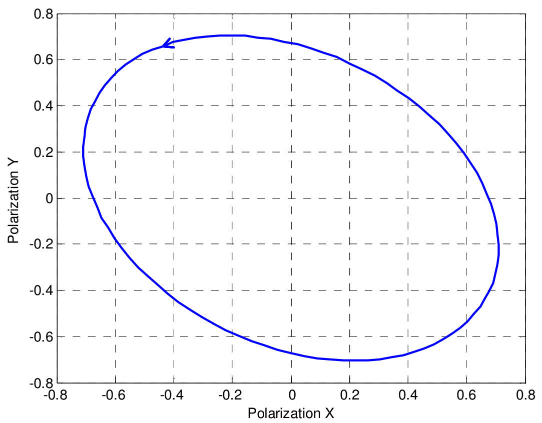

Figure 14. Polarization state of output photon for PMD=0.2 ps/km0.5

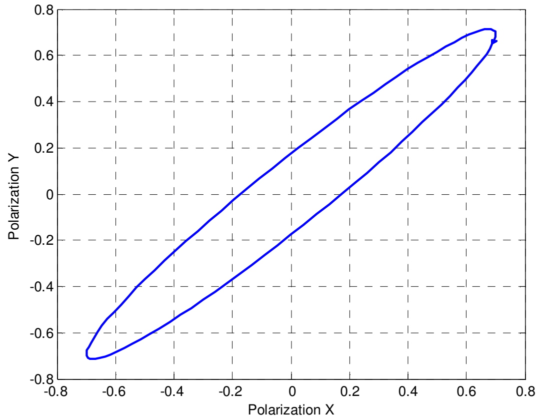

Figure 15. Polarization state of output photon for PMD=0.4 ps/km0.5

V. CONCLUSION

The effects of fiber-based channel on quantum state were analyzed in quantum communication. Depolarization of the photons in fibers was explained and analyzed. It was shown that the visibility of polarization of photon exponentially decrease by fiber channel length. Simulation results show that PMD is a critical parameter. PMD changed the polarization state and then destroyed the initial quantum state. The value of PMD in wavelength 1.55 µm is 0.4ps/√km. this value of PMD leads to change the initial polarization from horizontal to elliptical. Long distance quantum communication based on fiber channel suffers from this phenomenon. Quantum repeater can overcome decoherence problem by dividing distance to smaller segment and some protocols such as quantum purification and swapping.

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